Conservation of Relationality

On Stable Finite Reentry And The Taxation Of Public Form

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Pulse

Most conservation laws are written after a public world has already formed. Energy, charge, momentum, probability, and information are mature ledgers inside regimes where objects, fields, measurements, and comparison procedures are already available. This paper asks what must be conserved before such ledgers can exist.

The proposed answer is *relationality*: the comparison structure by which distinctions remain distinctions under lawful return. General Geometry begins not from primitive objects, points, or particles, but from possible distinguishability under finite hold. The first analytic discipline of such distinguishability is a closed positive comparison form

$$\mathcal{E}(f,g) = \langle Wf, Wg\rangle ,$$

where

$$W = L^{1/2}$$

is the first-order carrier associated to the nonnegative comparison burden operator L. The exact continuation flow

$$\mathcal{F}_s = e^{-isW}$$

preserves the comparison form:

$$\mathcal{E}(\mathcal{F}_s f, \mathcal{F}_s g) = \mathcal{E}(f,g).$$

This is the analytic form of conservation of relationality.

Finite public worlds do not receive the carrier as public objecthood directly. A cut first holds a quote

$$\mathbb{O}[\chi].$$

That held quote is normalized or compressed into finite participation

$$\mathbb{P}[\chi] = \mathrm{Norm}_{[0,1]}(\mathbb{O}[\chi]) , \qquad 0 \le \mathbb{P}[\chi] \le I.$$

The public projector extracted from participation is

$$\Pi[\mathbb{P}[\chi]] = \mathbf{1}_{(1/2,1]}(\mathbb{P}[\chi]) .$$

Thus the local public ladder is

$$\mathbb{O}[\chi] \longrightarrow \mathbb{P}[\chi] \longrightarrow \Pi[\mathbb{P}[\chi]].$$

The return tax is paid by participation. For a generic finite participation state

$$0 \le P \le I,$$

define

$$\mathfrak{N}_W(P) = \frac{1}{2} \left\| [W, P] \right\|_{\mathrm{HS}}^2 .$$

If

$$P = \sum_a \lambda_a \Pi_a$$

is the spectral decomposition of P, then

$$\mathfrak{N}_W(P) = \sum_{a<b} (\lambda_a - \lambda_b)^2 \left\| \Pi_a W \Pi_b \right\|_{\mathrm{HS}}^2 .$$

Burden therefore appears precisely where continuation crosses unequally held sectors. Specializing to a cut gives $P = \mathbb{P}[\chi]$.

Public thinghood is the sharp limit of finite participation. The canonical public projector is

$$\Pi[P] = \mathbf{1}_{(1/2,1]}(P),$$

and the sharpness defect

$$\delta_{\mathrm{sh}}(P) = \mathrm{tr}(P - P^2)$$

controls distance to public projectorhood:

$$\|P - \Pi[P]\|_{\mathrm{HS}}^2 \leq \delta_{\mathrm{sh}}(P).$$

A public object or stage is then not a single projector, but a transport-stable family of projectors across finite cuts:

$$\Pi[\mathbb{P}[\chi']] \approx T_{\chi \to \chi'} \Pi[\mathbb{P}[\chi]] T_{\chi \to \chi'}^* .$$

The resulting doctrine is simple. Exact carrierhood conserves relationality. Finite public participation pays for misalignment with that conservation. The taxes of failed stable return appear downstream as burden, mass-like behavior, holonomy, heat, hidden invariant shadows, dark remainders, semantic surprise, or codebook extension. Matter is therefore not primitive substance; it is sharpened, glued, burdened relationality. Conservation of relationality is the deep law. Matter is what happens when finite public surfaces fail to conserve it for free.

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1 Notation covenant

The main discipline of this paper is to keep held quote, participation, and public projectorhood distinct. Conservation belongs to exact carrierhood. The return tax is paid by finite participation.

The deep or prepublic coherence object is

$$\mathbb{O}.$$

A finite cut χ holds a quote

$$\mathbb{O}[\chi].$$

The corresponding finite participation state is

$$\mathbb{P}[\chi] = \mathrm{Norm}_{[0,1]}(\mathbb{O}[\chi])\,, \qquad 0 \leq \mathbb{P}[\chi] \leq I.$$

The public projector extracted from participation is

$$\Pi[\mathbb{P}[\chi]] = \mathbf{1}_{(1/2,1]}(\mathbb{P}[\chi])\,.$$

Thus the canonical local ladder is

$$\mathbb{O}[\chi] \longrightarrow \mathbb{P}[\chi] \longrightarrow \Pi[\mathbb{P}[\chi]].$$

The rungs are not interchangeable:

$$\mathbb{O}[\chi] \neq \mathbb{P}[\chi], \qquad \mathbb{P}[\chi] \neq \Pi[\mathbb{P}[\chi]]$$

in general.

When stating generic theorems, the paper uses

$$0 \leq P \leq I$$

for an arbitrary finite participation state. After proving the theorem, one may specialize to

$$P = \mathbb{P}[\chi].$$

Participation diagnostics are applied to P or $\mathbb{P}[\chi]$, not directly to the held quote:

$$\mathrm{part}(P) = \mathrm{tr}\, P,$$
$$\delta_{\mathrm{sh}}(P) = \mathrm{tr}(P - P^2),$$
$$\mathfrak{N}_W(P) = \frac{1}{2}\, \|[W, P]\|_{\mathrm{HS}}^2\,.$$

Trace-normalized held quotes and trace-normalized participation are also distinct:

$$\widehat{\mathbb{O}}[\chi] = \frac{\mathbb{O}[\chi]}{\mathrm{tr}\, \mathbb{O}[\chi]},$$

when defined, while

$$\widehat{\mathbb{P}}[\chi] = \frac{\mathbb{P}[\chi]}{\mathrm{tr}\, \mathbb{P}[\chi]},$$

when defined.

The compression is:

held quote is not participation;

participation is not projectorhood;

one local projector is not public worldhood;

publicity glues sharpened participation across cuts.

2 Introduction: conservation before conserved quantities

Most conservation laws are written after a world has already become public. This paper asks what must remain stable before energy, charge, momentum, probability, information, or matter can appear as public ledgers.

A mature physical theory usually begins after public objects, fields, measurements, and symmetries are already available. It then identifies conserved quantities inside that regime: energy, charge, momentum, probability, entropy, action, flux, or information. These are genuine and powerful invariants, but they are not the first invariants. They are public ledgers of a world that has already learned how to hold distinctions.

General Geometry begins one step earlier. It asks what must be stable enough for any finite world to have objects, particles, dimensions, meanings, laws, or conserved quantities at all.

The answer begins with Gradiance.

Definition 2.1 (Gradiance)**.** *Gradiance* is the prepublic availability of distinguishability: the minimal fact that not everything is held in total indifference. It is the possibility of contrast, leaning, direction, variation, or difference before any particular public object, coordinate system, particle, number, dimension, or meaning has been selected.

Gradiance is not itself a space of points, a primitive substance, a metric geometry, or an object among objects. It is the deeper condition that makes comparison possible at all.

The first finite mathematical shadow of Gradiance is not an object. It is a comparison law. In an analytic lane, this appears as a closed positive comparison form

$$\mathcal{E}.$$

Thus $\mathcal{E}$ is not Gradiance itself. It is a finite public readout of Gradiance: a disciplined way of saying which distinctions can be held together coherently enough to matter.

The first descent is therefore

$$\text{Gradiance} \longrightarrow \text{comparison} \longrightarrow \text{carrier} \longrightarrow \text{return}.$$

Relationality begins once distinguishability is not merely possible, but returnable.

Definition 2.2 (Relationality)**.** *Relationality* is distinguishability stabilized under comparison and return. It is the structure by which differences remain differences strongly enough to be held, carried, reencountered, compared, sharpened, and glued across finite cuts.

A finite world does not need every possible distinction to return. Most possible distinctions never become public. The claim is narrower: whatever becomes a world must preserve enough relational structure under return to be held again. If no distinguishability survives return, no public object forms, no measurement can be repeated, no particle can be identified, no dimension can stabilize, and no law can be stated.

Thus the core principle is:

before conserved quantities, there is conserved relationality.

The phrase *conservation of relationality* names the fact that exact carrierhood preserves the comparison structure of possible distinctions. The carrier does not first preserve a particle, a point, a word, or a number. It preserves the law by which differences remain comparable.

The first analytic form of this principle is simple. Let $\mathcal{E}$ be a closed positive comparison form and let

$$W = L^{1/2}$$

be the first-order carrier associated to its burden operator L. The exact continuation flow is

$$\mathcal{F}_s = e^{-isW}.$$

Since $\mathcal{F}_s$ is generated by the same carrier W, it preserves comparison:

$$\mathcal{E}(\mathcal{F}_s f, \mathcal{F}_s g) = \mathcal{E}(f, g).$$

This is the exact conservation law at the carrier level.

Finite public worlds, however, do not live as exact carrierhood alone. They live as finite quotes, finite participation states, sharpened projectors, local stages, and transport-stable gluing. A cut first holds

$$\mathbb{O}[\chi].$$

That held quote is normalized or compressed into participation:

$$\mathbb{P}[\chi] = \mathrm{Norm}_{[0,1]}(\mathbb{O}[\chi]), \qquad 0 \le \mathbb{P}[\chi] \le I.$$

A finite participation state may be real, stable, and public-facing without commuting perfectly with exact continuation. That mismatch is measured by continuation nonclosure.

For a generic finite participation state

$$0 \le P \le I,$$

define

$$\mathfrak{N}_W(P) = \frac{1}{2} \left\| [W, P] \right\|_{\mathrm{HS}}^2.$$

This is the first return tax. It measures how much finite participation pays to be carried by a continuation law with which it does not commute.

The central identity of the paper states that if

$$P = \sum_a \lambda_a \Pi_a,$$

then

$$\mathfrak{N}_W(P) = \sum_{a<b} (\lambda_a - \lambda_b)^2 \left\| \Pi_a W \Pi_b \right\|_{\mathrm{HS}}^2.$$

Thus burden appears precisely where continuation crosses sectors with unequal participation strengths.

This identity is the technical spine of the paper. It says that matter-like burden, holonomy, heat, semantic surprise, hidden invariant shadows, and codebook extension are not arbitrary additions. They are downstream forms of failed free return.

Exact carrierhood conserves relationality. Finite public participation pays for mismatch.

$$\text{exact carrierhood conserves relationality;} \qquad \text{finite participation pays for return.}$$

2.1 Finite reentry and public form

A finite world cannot hold all possible distinctions equally. It must quote. It must select some distinctions, defer others, and repeatedly maintain a surface on which the selected distinctions can return.

A held quote is not automatically public. A participation state is not automatically public. A local projector is not automatically public. Public form requires three further achievements: sharpening, carrier-compatibility, and gluing.

The paper uses three exact diagnostics.

First, finite participation is a positive contraction:

$$0 \leq P \leq I.$$

Second, its sharpness defect is

$$\delta_{\mathrm{sh}}(P) = \mathrm{tr}(P - P^2).$$

This vanishes exactly when P is a projector.

Third, its continuation nonclosure is

$$\mathfrak{N}_W(P) = \frac{1}{2} \, \|[W, P]\|_{\mathrm{HS}}^2 \, .$$

Together these distinguish three local regimes:

$$\begin{array}{rcl} \text{unresolved} & : & \delta_{\mathrm{sh}}(P) \text{ large,} \\ \text{matter-like} & : & \delta_{\mathrm{sh}}(P) \text{ small but } \mathfrak{N}_W(P) > 0, \\ \text{messenger-like} & : & \delta_{\mathrm{sh}}(P) \text{ small and } \mathfrak{N}_W(P) \approx 0. \end{array}$$

This is the first local taxonomy of public emergence.

2.2 What is conserved

The conservation claimed here is not conservation of a fixed object inventory. It is not the claim that every local distinction survives as itself in every finite quote. It is subtler.

At the exact carrier level, what is conserved is the comparison form:

$$\mathcal{E}(f, g).$$

At the finite-public level, what is preserved only imperfectly is a selected participation-quote of that comparison structure.

The finite world can lose phase, average over continuation, sharpen participation, collapse to a public projector, or rebuild under heat. These operations may reduce what is visible, but they do not create distinguishability ex nihilo. They expose, compress, reorganize, or tax latent relational structure.

Thus:

finite public worlds do not create relationality;

they quote relationality under hold, sharpening, transport, and gluing.

When participation is aligned with exact continuation, it is carried freely. When it is misaligned, it pays nonclosure.

2.3 Relation to the General Geometry stack

This paper is written inside General Geometry. It assumes the basic vocabulary of finite distinguishability, comparison forms, first-order carrier laws, finite participation states, public projector extraction, continuation nonclosure, and public gluing.

It does not attempt to rederive the full public-stage theorem, the response-complete $1+2+3$ grammar, the representation shell package, the dark-sector normal form, or Semantic Mechanics. Those appear elsewhere in the General Geometry stack.

The present paper extracts the common law beneath them:

stable finite reentry is possible only where relationality can be conserved or lawfully taxed.

This gives a compact principle from which many later phenomena can be read as descendants.

2.4 Status of claims

The paper has three kinds of statements.

The first kind is exact mathematics: spectral calculus, unitary preservation of comparison energy, commutator identities, nearest-projector extraction, and gluing estimates.

The second kind is General Geometry doctrine: finite public worlds are sharpened and glued participation-quotes of deeper distinguishability.

The third kind is interpretive consequence: matter, mass, holonomy, dark remainder, and meaning are different public reports of failed free return.

The exact claims are deliberately modest. The interpretive reach is large because the same exact mechanism keeps reappearing:

$$\text{carrier} \longrightarrow \text{finite participation} \longrightarrow \text{commutator} \longrightarrow \text{return tax.}$$

3 Comparison before objects

The first analytic object is not a space of points. It is a law of comparison. Points, particles, dimensions, and meanings are later public codewords of stable distinguishability.

General Geometry begins with possible distinguishability rather than with already-public objects. To distinguish anything, a finite regime must be able to compare. Comparison is therefore earlier than metric distance.

Let $\mathscr{H}$ be a complex Hilbert space of distinguishability profiles. At this level, an element

$$f \in \mathscr{H}$$

need not be interpreted as a function on a primitive manifold. It is a possible profile of difference.

Definition 3.1 (Comparison form)**.** A *comparison form* on $\mathscr{H}$ is a densely defined, closed, nonnegative sesquilinear form

$$\mathcal{E} : \mathcal{D}(\mathcal{E}) \times \mathcal{D}(\mathcal{E}) \to \mathbb{C}$$

satisfying

$$\mathcal{E}(f, f) \geq 0 \qquad (f \in \mathcal{D}(\mathcal{E})).$$

We write

$$\mathcal{E}[f] := \mathcal{E}(f, f).$$

The value $\mathcal{E}[f]$ is the comparison cost of the profile f. It measures the burden of holding the variations represented by f.

A comparison form is the first disciplined mathematical shadow of Gradiance. It does not yet say what the world is made of. It says which differences can be held together coherently enough to matter.

3.1 The burden operator

By the representation theorem for closed positive forms, there exists a unique nonnegative self-adjoint operator

$$L \geq 0$$

such that

$$\mathcal{D}(\mathcal{E}) = \mathcal{D}(L^{1/2})$$

and

$$\mathcal{E}(f, g) = \left\langle L^{1/2} f, L^{1/2} g \right\rangle .$$

The operator L is the public squared-burden operator of comparison.

Definition 3.2 (First-order carrier)**.** The first-order carrier associated to $\mathcal{E}$ is

$$W := L^{1/2}.$$

Thus

$$\mathcal{E}(f, g) = \langle Wf, Wg \rangle .$$

The central inversion is:

the second-order burden L is the square of the first-order carrier W.

In a Riemannian public envelope one has

$$L = -\Delta_g, \qquad W = \sqrt{-\Delta_g}.$$

But this is a specialization, not the starting point. The comparison form comes first. The metric and Laplacian are mature public readouts.

3.2 The carrier triangle

Once W exists, three canonical flows appear:

$$\mathcal{P}_\sigma = e^{-\sigma W},$$

$$\mathcal{F}_s = e^{-isW},$$

and

$$\mathcal{H}_r = e^{-rW^2}.$$

These are, respectively, finite holding, exact continuation, and rebuild under squared burden.

They are not three unrelated mechanisms. They are three functional-calculus faces of one first-order carrier:

$$\mathcal{P}_\sigma = e^{-\sigma W}, \qquad \mathcal{F}_s = e^{-isW}, \qquad \mathcal{H}_r = e^{-rW^2}.$$

Finite hold is real carrier damping. Exact continuation is imaginary carrier flow. Heat rebuild is squared-burden relaxation.

3.3 No primitive object inventory

The comparison-carrier sequence is:

$$\mathcal{E} \longrightarrow L \longrightarrow W = L^{1/2} \longrightarrow \mathcal{P}_\sigma,\ \mathcal{F}_s,\ \mathcal{H}_r.$$

There is no primitive object inventory in this sequence. Objects appear later, when finite participation sharpens toward projectors and glues across cuts.

This is the first sense in which conservation of relationality precedes ordinary conservation laws. Before energy or charge can be recorded, distinctions must be comparable enough to return.

relationality is comparison structure before public objecthood.

4 Exact return and conservation of relationality

Exact continuation is the return law that preserves comparison. This is the analytic form of conservation of relationality.

The exact continuation flow is

$$\mathcal{F}_s = e^{-isW}.$$

Since W is self-adjoint, $\mathcal{F}_s$ is unitary. It also commutes with W, since both are functions of W.

Theorem 4.1 (Carrier conservation of comparison)**.** *For all $f, g \in \mathcal{D}(W)$ and all $s \in \mathbb{R}$,*

$$\mathcal{E}(\mathcal{F}_s f, \mathcal{F}_s g) = \mathcal{E}(f, g).$$

In particular,

$$\mathcal{E}[\mathcal{F}_s f] = \mathcal{E}[f].$$

Proof. Using $\mathcal{E}(f,g) = \langle Wf, Wg\rangle$, unitarity of $\mathcal{F}_s$, and commutation of W with $\mathcal{F}_s$, we have

$$\mathcal{E}(\mathcal{F}_s f, \mathcal{F}_s g) = \langle W\mathcal{F}_s f, W\mathcal{F}_s g\rangle = \langle \mathcal{F}_s Wf, \mathcal{F}_s Wg\rangle = \langle Wf, Wg\rangle = \mathcal{E}(f,g).$$

The quadratic statement follows by taking $g = f$. □

This theorem is the basic analytic statement of conservation of relationality:

exact carrier flow preserves comparison.

It is not yet conservation of energy in a physical spacetime. It is earlier. It is the preservation of the structure by which differences remain comparable under return.

4.1 Noether form

The same principle has a familiar commutant form.

Let ρ be a trace-class state or finite-dimensional density operator and let

$$\rho_s = \mathcal{F}_s \rho \mathcal{F}_s^*.$$

Let A be an observable or test operator.

Proposition 4.2 (Commutant conservation)**.** *If*

$$[A, W] = 0,$$

then

$$\mathrm{tr}(\rho_s A) = \mathrm{tr}(\rho A)$$

for all s.

Proof. Since A commutes with W, it commutes with $\mathcal{F}_s = e^{-isW}$. Therefore

$$\mathrm{tr}(\rho_s A) = \mathrm{tr}(\mathcal{F}_s \rho \mathcal{F}_s^* A) = \mathrm{tr}(\rho \mathcal{F}_s^* A \mathcal{F}_s) = \mathrm{tr}(\rho A).$$

□

Thus conserved public ledgers are late expressions of carrier commutation:

$$[A, W] = 0 \quad \Longrightarrow \quad A \text{ is conserved under exact return.}$$

This is the Noether-style shadow of conservation of relationality.

4.2 Holding and rebuild are not violations

The holding flow

$$\mathcal{P}_\sigma = e^{-\sigma W}$$

and rebuild flow

$$\mathcal{H}_r = e^{-rW^2}$$

do not preserve comparison energy in general. They contract it:

$$\mathcal{E}[\mathcal{P}_\sigma f] \le \mathcal{E}[f],$$

$$\mathcal{E}[\mathcal{H}_r f] \le \mathcal{E}[f].$$

This is not a contradiction. Holding and rebuild are finite-access operations. They are not exact return. They are public envelopes of carrierhood under finite hold and squared burden.

Exact continuation preserves. Holding quotes. Heat rebuilds.

$$\text{continuation preserves;} \qquad \text{hold and rebuild finite-quote.}$$

The conservation law lives at the exact carrier level. Finite public worlds live by controlled departures from it.

4.3 Relationality versus information

Conservation of relationality is not identical to conservation of information. Information is usually defined relative to an already selected coding, probability, entropy, or message grammar. Relationality is earlier. It is the comparison structure that makes such codes possible.

A finite world may lose information in one public codebook while preserving relationality at the deeper carrier level. Conversely, a public codebook may preserve many symbols while losing the relational structure that made them meaningful.

Thus:

$$\text{information is codebook-relative;} \qquad \text{relationality is comparison-relative.}$$

Conservation of relationality is the statement that exact carrierhood preserves the comparison law from which codebooks may later be extracted.

5 Finite participation and the return tax

A finite cut holds a quote, but the return tax is paid by participation. Finite participation can be carried freely only when it aligns with exact continuation.

A finite public world does not hold the whole comparison carrier. It holds finite quotes and derives participation from them:

$$\mathbb{O}[\chi] \longrightarrow \mathbb{P}[\chi].$$

Throughout this section, let $\mathcal{K}$ be a finite-dimensional active carrier space and let

$$W = W^*$$

be the local first-order carrier.

Definition 5.1 (Finite participation state)**.** A *finite participation state* is a positive contraction

$$0 \le P \le I$$

on $\mathcal{K}$.

The eigenvalues of P are participation strengths. They are not first probabilities. They record how strongly sectors of distinguishability are admitted into the finite regime.

At a cut χ, the participation state is

$$P = \mathbb{P}[\chi].$$

Finite participation is return-stable under the carrier exactly when

$$[W, P] = 0.$$

When this fails, participation pays a continuation tax.

Definition 5.2 (Return tax / continuation nonclosure)**.** The return tax of P relative to W is

$$\mathfrak{T}_W(P) := \mathfrak{N}_W(P) = \frac{1}{2} \left\| [W, P] \right\|_{\mathrm{HS}}^2 .$$

This scalar is nonnegative and vanishes exactly when P commutes with W.

Thus:

$$\mathfrak{N}_W(P) = 0 \quad \Longleftrightarrow \quad P \text{ is carrier-aligned.}$$

5.1 The spectral return-tax identity

Let

$$P = \sum_a \lambda_a \Pi_a$$

be the spectral decomposition of P, where

$$0 \le \lambda_a \le 1.$$

Theorem 5.3 (Return-tax identity)**.** *For every finite participation state P,*

$$\mathfrak{N}_W(P) = \sum_{a<b} (\lambda_a - \lambda_b)^2 \left\| \Pi_a W \Pi_b \right\|_{\mathrm{HS}}^2 .$$

Equivalently,

$$\mathfrak{N}_W(P) = \frac{1}{2} \sum_{a,b} (\lambda_a - \lambda_b)^2 \left\| \Pi_a W \Pi_b \right\|_{\mathrm{HS}}^2 .$$

Proof. Using the spectral decomposition of P,

$$WP = \sum_{a,b} \lambda_b \Pi_a W \Pi_b,$$

while

$$PW = \sum_{a,b} \lambda_a \Pi_a W \Pi_b.$$

Therefore

$$[W, P] = \sum_{a,b} (\lambda_b - \lambda_a) \Pi_a W \Pi_b.$$

The Hilbert–Schmidt orthogonality of distinct spectral blocks gives

$$\left\| [W, P] \right\|_{\mathrm{HS}}^2 = \sum_{a,b} (\lambda_a - \lambda_b)^2 \left\| \Pi_a W \Pi_b \right\|_{\mathrm{HS}}^2 .$$

Multiplying by $1/2$ gives the second formula.

Since $W = W^*$, the terms (a, b) and (b, a) have equal Hilbert–Schmidt norms. The diagonal terms vanish. Pairing the off-diagonal terms gives the first formula. □

This identity is the central theorem of the paper.

It says:

> burden appears precisely where continuation crosses unequally held sectors.

If two sectors have equal participation strength, continuation between them creates no return tax. If their participation strengths differ, carrier coupling between them is taxed in proportion to the square of the participation contrast.

5.2 Carrier-basis form

The same tax has a second exact expression.

Let

$$We_\alpha = \omega_\alpha e_\alpha$$

be a W-eigenbasis. Write

$$P_{\alpha\beta} = \langle Pe_\beta, e_\alpha \rangle .$$

Then

$$[W, P]_{\alpha\beta} = (\omega_\alpha - \omega_\beta) P_{\alpha\beta}.$$

Proposition 5.4 (Carrier-basis tax)**.** *In a W-eigenbasis,*

$$\mathfrak{N}_W(P) = \frac{1}{2}\sum_{\alpha,\beta}(\omega_\alpha - \omega_\beta)^2 |P_{\alpha\beta}|^2.$$

Proof. The displayed matrix-entry identity gives

$$\|[W,P]\|_{\mathrm{HS}}^2 = \sum_{\alpha,\beta}(\omega_\alpha - \omega_\beta)^2 |P_{\alpha\beta}|^2.$$

Multiplying by 1/2 proves the claim. □

Thus the return tax has two faces:

participation basis	:	sector contrast times carrier coupling,
carrier basis	:	off-diagonal coherence between unequal carrier frequencies.

The same quantity measures both unequal participation crossing and carrier-incompatible coherence.

5.3 Dephasing as tax reduction

Define the dephasing flow

$$\mathcal{D}_t(P) = e^{-t\,\mathrm{ad}_W^2} P,$$

where

$$\mathrm{ad}_W(P) = [W,P].$$

Equivalently,

$$\dot{P}_t = -[W,[W,P_t]].$$

In a W-eigenbasis,

$$(P_t)_{\alpha\beta} = e^{-t(\omega_\alpha-\omega_\beta)^2}(P_0)_{\alpha\beta}.$$

Proposition 5.5 (Tax monotonicity under dephasing)**.** *If*

$$P_t = \mathcal{D}_t(P_0),$$

then

$$\frac{d}{dt}\mathfrak{N}_W(P_t) = -\,\|[W,[W,P_t]]\|_{\mathrm{HS}}^2 \le 0.$$

Proof. Using Proposition 5.4,

$$\mathfrak{N}_W(P_t) = \frac{1}{2}\sum_{\alpha,\beta}(\omega_\alpha - \omega_\beta)^2 e^{-2t(\omega_\alpha-\omega_\beta)^2}|(P_0)_{\alpha\beta}|^2.$$

Differentiating termwise gives

$$\frac{d}{dt}\mathfrak{N}_W(P_t) = -\sum_{\alpha,\beta}(\omega_\alpha - \omega_\beta)^4 |(P_t)_{\alpha\beta}|^2.$$

But

$$[W,[W,P_t]]_{\alpha\beta} = (\omega_\alpha - \omega_\beta)^2 (P_t)_{\alpha\beta}.$$

Hence

$$\frac{d}{dt}\mathfrak{N}_W(P_t) = -\,\|[W,[W,P_t]]\|_{\mathrm{HS}}^2\,.$$

□

Dephasing is the canonical flow toward carrier alignment. It strips away off-diagonal coherence between unequal continuation frequencies.

Thus:

dephasing reduces the return tax by suppressing carrier-incompatible coherence.

5.4 Distance to carrier alignment

Let
$$W = \sum_\alpha \omega_\alpha E_\alpha$$
be the spectral decomposition of W. Let $\mathcal{E}_W$ denote the Hilbert–Schmidt orthogonal projection onto the commutant of W:
$$\mathcal{E}_W(A) = \sum_\alpha E_\alpha A E_\alpha.$$

Assume the finite active carrier spectrum has positive gap
$$\Delta_W = \min_{\omega_\alpha \neq \omega_\beta} |\omega_\alpha - \omega_\beta| > 0.$$

Proposition 5.6 (Tax controls distance to the carrier commutant)**.** *If* $\Delta_W > 0$*, then*
$$\mathfrak{N}_W(P) \geq \frac{\Delta_W^2}{2} \left\| P - \mathcal{E}_W(P) \right\|_{\mathrm{HS}}^2 .$$

Proof. In the W-eigenbasis,
$$P - \mathcal{E}_W(P)$$
consists exactly of the off-diagonal blocks with unequal W-eigenvalues. Therefore
$$\|P - \mathcal{E}_W(P)\|_{\mathrm{HS}}^2 = \sum_{\omega_\alpha \neq \omega_\beta} |P_{\alpha\beta}|^2.$$

By Proposition 5.4,
$$\mathfrak{N}_W(P) = \frac{1}{2} \sum_{\omega_\alpha \neq \omega_\beta} (\omega_\alpha - \omega_\beta)^2 |P_{\alpha\beta}|^2 \geq \frac{\Delta_W^2}{2} \sum_{\omega_\alpha \neq \omega_\beta} |P_{\alpha\beta}|^2.$$

This is the desired bound. □

Thus small return tax implies proximity to the carrier commutant, provided the active carrier spectrum is gapped.

low tax means near carrier alignment.

6 Public thinghood by sharpening

Finite participation becomes public thinghood by sharpening. The half-threshold is not arbitrary; it is the nearest-projector boundary between public zero and public one.

A finite participation state

$$0 \le P \le I$$

is generally soft. Its spectrum records participation strengths between zero and one. A public object, by contrast, is projector-like: a sector is in or out.

This section records the exact sharpening facts needed for public thinghood.

Definition 6.1 (Sharpness defect)**.** The sharpness defect of a finite participation state P is

$$\delta_{\mathrm{sh}}(P) = \mathrm{tr}(P - P^2).$$

If

$$P = \sum_a \lambda_a \Pi_a,$$

then

$$\delta_{\mathrm{sh}}(P) = \sum_a \lambda_a (1 - \lambda_a)\,\mathrm{rank}(\Pi_a).$$

Thus

$$\delta_{\mathrm{sh}}(P) = 0 \quad \Longleftrightarrow \quad P^2 = P.$$

Sharpness defect is the exact defect of projectorhood.

6.1 Canonical public projector

Definition 6.2 (Canonical public projector)**.** The canonical public projector extracted from finite participation P is

$$\Pi[P] = \mathbf{1}_{(1/2,1]}(P).$$

The threshold $1/2$ is the boundary between being closer to public zero and being closer to public one.

Theorem 6.3 (Nearest-projector theorem)**.** *Let $\mathcal{P}(\mathcal{K})$ be the set of orthogonal projectors on $\mathcal{K}$. Then $\Pi[P]$ is a Hilbert–Schmidt nearest projector to P:*

$$\|P - \Pi[P]\|_{\mathrm{HS}} = \min_{\Pi \in \mathcal{P}(\mathcal{K})} \|P - \Pi\|_{\mathrm{HS}},$$

up to ambiguity on the $1/2$-eigenspace. More explicitly,

$$\|P - \Pi[P]\|_{\mathrm{HS}}^2 = \sum_a \min(\lambda_a^2, (1 - \lambda_a)^2)\,\mathrm{rank}(\Pi_a).$$

Proof. Diagonalize P in an orthonormal eigenbasis. For any projector Π,

$$\|P - \Pi\|_{\mathrm{HS}}^2 = \mathrm{tr}(P^2) + \mathrm{tr}(\Pi) - 2\,\mathrm{tr}(P\Pi).$$

For fixed rank, minimizing this is equivalent to maximizing $\mathrm{tr}(P\Pi)$, achieved by projecting onto the largest eigenvalues. Allowing the rank to vary, each eigenvalue λ is assigned either to public zero at cost λ^2 or to public one at cost $(1 - \lambda)^2$. The latter is smaller exactly when

$$\lambda > \frac{1}{2}.$$

Thus the nearest projector is the spectral projector onto eigenvalues greater than 1/2, up to equality at 1/2. □

Thus:

$$\frac{1}{2} = \text{the canonical public decision seam.}$$

At a finite cut χ, the local public projector is therefore

$$\Pi[\mathbb{P}[\chi]] = \mathbf{1}_{(1/2,1]}\big(\mathbb{P}[\chi]\big).$$

6.2 Sharpness controls public thinghood

Corollary 6.4 (Sharpness defect bound)**.** *For every finite participation state P,*

$$\|P - \Pi[P]\|_{\mathrm{HS}}^2 \le \delta_{\mathrm{sh}}(P).$$

Proof. For every $0 \le \lambda \le 1$,

$$\min(\lambda^2, (1-\lambda)^2) \le \lambda(1-\lambda).$$

Summing over the spectrum proves the result. □

So small sharpness defect implies closeness to a public projector:

$$\delta_{\mathrm{sh}}(P) \ll 1 \quad \Longrightarrow \quad P \text{ is near public thinghood.}$$

6.3 Public rank from participation

Define the public rank

$$N_{\mathrm{pub}}(P) = \mathrm{rank}(\Pi[P]).$$

Corollary 6.5 (Participation count approximates public rank)**.** *For every finite participation state P,*

$$|\mathrm{tr}(P) - N_{\mathrm{pub}}(P)| \le 2\delta_{\mathrm{sh}}(P).$$

Proof. For every $0 \le \lambda \le 1$,

$$\big|\lambda - \mathbf{1}_{\lambda>1/2}\big| \le 2\lambda(1-\lambda).$$

Summing over eigenvalues gives the estimate. □

Thus public dimension is a sharpened count of participation:

continuous participation becomes integer public rank when sharpness defect is small.

6.4 Thinghood, burden, and messengerhood

The two scalar diagnostics

$$\delta_{\mathrm{sh}}(P) \qquad \text{and} \qquad \mathfrak{N}_W(P)$$

separate public regimes.

Pre-public regime. If $\delta_{\mathrm{sh}}(P)$ is large, then no robust public projector has formed.

Sharpened but burdened regime. If

$$\delta_{\mathrm{sh}}(P) \ll 1$$

but

$$\mathfrak{N}_W(P) > 0,$$

then participation has become object-like but is not freely carried. This is the structural regime of matter-like burden.

Sharpened messenger regime. If

$$\delta_{\mathrm{sh}}(P) \ll 1$$

and

$$\mathfrak{N}_W(P) \approx 0,$$

then participation is object-like and approximately carrier-aligned. This is the structural regime of messengerhood.

Thus:

$$\text{thinghood} = \text{near projectorhood},$$
$$\text{matterhood} = \text{near projectorhood plus nonzero return tax},$$
$$\text{messengerhood} = \text{near projectorhood plus low return tax}.$$

6.5 Near-projector transport stability

Let X be a bounded transport generator. If X preserves the public projector, then it nearly preserves the unresolved participation state when sharpness defect is small.

Proposition 6.6 (Near-projector transport stability)**.** *Suppose*

$$[X, \Pi[P]] = 0.$$

Then

$$\|[X, P]\|_{\mathrm{HS}} \le 2\,\|X\|_{\mathrm{op}} \sqrt{\delta_{\mathrm{sh}}(P)}.$$

Proof. Since $[X, \Pi[P]] = 0$,

$$[X, P] = [X, P - \Pi[P]].$$

Therefore

$$\|[X, P]\|_{\mathrm{HS}} \le 2\,\|X\|_{\mathrm{op}}\,\|P - \Pi[P]\|_{\mathrm{HS}}\,.$$

By Corollary 6.4,

$$\|P - \Pi[P]\|_{\mathrm{HS}} \le \sqrt{\delta_{\mathrm{sh}}(P)}.$$

Combining the estimates proves the claim. □

This is the first exact approximate-gauge statement of the paper:

symmetries of the sharpened public projector are approximate symmetries of unresolved participation.

6.6 Cut-specific reading

At a cut χ, the generic participation state is

$$P = \mathbb{P}[\chi].$$

The public projector is

$$\Pi_\chi := \Pi[\mathbb{P}[\chi]].$$

Thus the cut-local public ladder is

$$\mathbb{O}[\chi] \longrightarrow \mathbb{P}[\chi] \longrightarrow \Pi_\chi.$$

The held quote is not the participation state, and the participation state is not the projector. Public thinghood appears only after projector extraction from participation.

6.7 Summary

Finite public thinghood is sharpened participation. The half-threshold projector

$$\Pi[P] = \mathbf{1}_{(1/2,1]}(P)$$

is the Hilbert–Schmidt nearest public projector. Sharpness defect controls distance to projectorhood. Public matter-like behavior appears when participation is near-projector but pays nonzero return tax. Messenger-like behavior appears when participation is near-projector and low-tax.

7 Public worldhood by gluing

One local projector is not yet a public world. Publicity is gluing: stable transport of sharpened participation across finite cuts.

A finite participation state becomes thing-like when it sharpens toward projectorhood. But one local projector is still not a world. Publicity requires transport stability across finite cuts.

Let χ, χ' be nearby cuts. Let

$$\mathcal{K}_\chi, \qquad \mathcal{K}_{\chi'}$$

be their active carrier spaces, and let

$$T_{\chi\to\chi'} : \mathcal{K}_\chi \to \mathcal{K}_{\chi'}$$

be a comparison transport.

The full form of public gluing is

$$\Pi[\mathbb{P}[\chi']] \approx T_{\chi\to\chi'}\Pi[\mathbb{P}[\chi]]T^*_{\chi\to\chi'}.$$

This teaches the correct object: public gluing glues sharpened participation, not raw held quotes.

After this point, define the shorthand

$$\Pi_\chi := \Pi[\mathbb{P}[\chi]].$$

Then the gluing condition becomes

$$\Pi_{\chi'} \approx T_{\chi\to\chi'}\Pi_\chi T^*_{\chi\to\chi'}.$$

Definition 7.1 (Gluing defect)**.** The gluing defect between χ and χ' for a candidate public projector family Π is

$$\mathcal{G}(\chi, \chi'; \Pi) = \left\| \Pi_{\chi'} - T_{\chi\to\chi'} \Pi_\chi T^*_{\chi\to\chi'} \right\|_{\mathrm{HS}}.$$

A family of projectors glues over a region when the gluing defect remains controlled along neighboring cuts.

Thus:

public objecthood is transport-stable projectorhood across finite cuts.

7.1 Path gluing

Let

$$\gamma = (\chi_0, \chi_1, \ldots, \chi_N)$$

be a path of cuts. Let

$$T_\gamma = T_{\chi_{N-1}\to\chi_N} \cdots T_{\chi_0\to\chi_1}$$

be the ordered transport along the path.

Proposition 7.2 (Path gluing bound)**.** *Assume the transports preserve Hilbert–Schmidt norm on the relevant active subspaces. Then*

$$\left\| \Pi_{\chi_N} - T_\gamma \Pi_{\chi_0} T^*_\gamma \right\|_{\mathrm{HS}} \le \sum_{j=0}^{N-1} \mathcal{G}(\chi_j, \chi_{j+1}; \Pi).$$

Proof. Insert intermediate transported projectors and apply the triangle inequality. Each edge contributes its gluing defect, and norm preservation by the remaining tail transport keeps each defect unchanged. □

Thus local gluing controls long public travel:

stable public range is accumulated gluing control.

7.2 Public stage

The same idea applies to a public stage. A local public stage is represented by a projector

$$\Pi_{S,\chi}.$$

A public stage over a region is a family

$$\{\Pi_{S,\chi}\}$$

such that

$$\left\| \Pi_{S,\chi'} - T_{\chi\to\chi'} \Pi_{S,\chi} T^*_{\chi\to\chi'} \right\|_{\mathrm{HS}}$$

is small for nearby cuts.

Stage subprojectors are already public-stage pieces, so they are written directly:

$$\Pi_{\mathsf{L},\chi}, \qquad \Pi_{D,\chi}, \qquad \Pi_{C,\chi},$$

when the corresponding stage decomposition is present.

Their gluing is stage-projector gluing:

$$\Pi_{\mathsf{L},\chi'} \approx T_{\chi\to\chi'} \Pi_{\mathsf{L},\chi} T^*_{\chi\to\chi'}.$$

Thus:

public reality is gluable stagehood.

7.3 Holonomy defect

Let γ be a loop of cuts beginning and ending at χ_0. Let

$$T_\gamma$$

be the ordered transport around the loop.

Definition 7.3 (Loop nonclosure / holonomy defect)**.** The holonomy defect of a projector Π around γ is

$$\mathfrak{S}_\gamma(\Pi) = \left\| T_\gamma \Pi T_\gamma^* - \Pi \right\|_{\mathrm{HS}}.$$

If

$$\mathfrak{S}_\gamma(\Pi) = 0,$$

the distinction closes around the loop. If

$$\mathfrak{S}_\gamma(\Pi) > 0,$$

the distinction returns altered.

This is the finite-public source of force-like correction:

force is correction pressure induced by loop nonclosure.

The statement is structural. It does not replace a descendant force law. It identifies the common geometry beneath force laws: transport, return, mismatch, correction.

7.4 Stage-slip versus internal holonomy

There are two distinct loop questions.

First, the public stage projector itself may fail to return:

$$T_\gamma \Pi_{S,\chi_0} T_\gamma^* \neq \Pi_{S,\chi_0}.$$

This is stage-slip holonomy.

Second, the stage projector may return while sections inside the stage rotate:

$$T_\gamma \Pi_{S,\chi_0} T_\gamma^* = \Pi_{S,\chi_0},$$

but the induced internal transport

$$U_\gamma^S := \Pi_{S,\chi_0} T_\gamma \Pi_{S,\chi_0}\big|_{\operatorname{Ran}\Pi_{S,\chi_0}}$$

acts nontrivially on the returned stage:

$$U_\gamma^S \neq I_{\operatorname{Ran}\Pi_{S,\chi_0}}.$$

This is internal public-stage holonomy.

Thus:

stage-slip asks whether the subspace returns;

internal holonomy asks how sections rotate inside the returned subspace.

Both are forms of finite-public nonclosure.

7.5 Smooth projector fields

When the cut space has a smooth structure, let

$$x \mapsto \Pi_S(x)$$

be a smooth projector field:

$$\Pi_S^2 = \Pi_S = \Pi_S^*.$$

Differentiating

$$\Pi_S^2 = \Pi_S$$

gives

$$(d\Pi_S)\Pi_S + \Pi_S(d\Pi_S) = d\Pi_S.$$

Multiplying on both sides by Π_S gives

$$\Pi_S(d\Pi_S)\Pi_S = 0.$$

Thus projector variation is off-diagonal:

$$d\Pi_S = \Pi_S(d\Pi_S)(I - \Pi_S) + (I - \Pi_S)(d\Pi_S)\Pi_S.$$

A public stage can vary only by rotating into its complement.

The projected connection on the stage bundle is

$$\nabla^S = \Pi_S d.$$

Proposition 7.4 (Projected curvature)**.** *The curvature of ∇^S is*

$$F_S = \Pi_S(d\Pi_S \wedge d\Pi_S)\Pi_S.$$

Proof. Let u be a section of the public stage, so $\Pi_S u = u$. Then

$$\nabla^S u = \Pi_S du.$$

Applying ∇^S again,

$$(\nabla^S)^2 u = \Pi_S d(\Pi_S du) = \Pi_S(d\Pi_S \wedge du),$$

since $d^2 u = 0$. But

$$du = d(\Pi_S u) = (d\Pi_S)u + \Pi_S du.$$

The term involving $\Pi_S du$ vanishes because

$$\Pi_S(d\Pi_S)\Pi_S = 0.$$

Therefore

$$(\nabla^S)^2 u = \Pi_S(d\Pi_S \wedge d\Pi_S)u.$$

Since $u = \Pi_S u$, the curvature is

$$F_S = \Pi_S(d\Pi_S \wedge d\Pi_S)\Pi_S.$$

□

This is the smooth public-stage version of loop nonclosure:

curvature is infinitesimal holonomy of glued public stagehood.

7.6 Stable finite reentry

We can now state the working condition of stable finite reentry.

A distinction is stably reentered when:

1. it is held as a finite quote,
2. it becomes finite participation,
3. it sharpens toward projectorhood,
4. its return tax under W is controlled,
5. its projectors glue across cuts,
6. its loop holonomy is either small or correctable.

This is the finite-public form of conservation of relationality:

a finite world is stable when its quoted distinctions can return recognizably.

7.7 Summary

Public worldhood is not one local state. It is transport-stable projectorhood across cuts. The correct gluing object is sharpened participation:

$$\Pi[\mathbb{P}[\chi']] \approx T_{\chi\to\chi'}\Pi[\mathbb{P}[\chi]]T^*_{\chi\to\chi'}.$$

After defining

$$\Pi_\chi := \Pi[\mathbb{P}[\chi]],$$

the compact form is

$$\Pi_{\chi'} \approx T_{\chi\to\chi'}\Pi_\chi T^*_{\chi\to\chi'}.$$

Loop mismatch is holonomy. Holonomy produces correction pressure. Stable finite reentry requires held quote, participation, sharpening, controlled tax, gluing, and correctable holonomy.

8 The taxonomy of failed return

Finite public form fails to conserve relationality for free. The different ways it fails become familiar descendants: burden, heat, holonomy, hidden shadows, dark remainders, and semantic surprise.

The commutator

$$[W, P]$$

is the local analytic source of return failure for finite participation. But return failure has many public descendants depending on the lane, the codebook, and the stage.

This section gives a compact taxonomy.

8.1 Burden and mass-like behavior

When finite participation is sharp enough to be object-like but fails to commute with continuation,

$$\delta_{\mathrm{sh}}(P) \ll 1, \qquad \mathfrak{N}_W(P) > 0,$$

it has public form but not free return.

This is the structural regime of mass-like burden.

mass-like behavior is sharpened finite form with nonzero return tax.

In descendant particle theory, this becomes shell mass, stage burden, Higgs-like nonclosure, and coupling to local public-stage misalignment. The present paper does not need those details. The core mechanism is already present in the commutator tax.

8.2 Heat and dephasing

When finite participation carries off-diagonal coherence in the carrier basis, dephasing reduces the return tax:

$$\dot{P}_t = -[W, [W, P_t]].$$

This suppresses coherence between unequal carrier frequencies.

Thus:

heat-like rebuild is the public relaxation of carrier-incompatible coherence.

At the profile level this appears as

$$\mathcal{H}_r = e^{-rW^2}.$$

At the participation-state level it appears as

$$e^{-t\,\mathrm{ad}_W^2}.$$

Both are squared-burden rebuilds.

8.3 Holonomy and force

When a public projector is transported around a loop and returns altered,

$$\mathfrak{S}_\gamma(\Pi) > 0,$$

the system must either accept the mismatch, rebuild its model, or apply correction.

This is the structural source of force-like behavior:

force is correction pressure from public-stage nonclosure.

In gauge descendants, the same geometry appears as curvature. In semantic descendants, it appears as surprise. In physical public-stage descendants, it appears as force and constraint.

8.4 Hidden invariant shadows

Sometimes a public codebook cannot split a hidden channel system. The hidden channels may become visible only after codebook extension, while the original public codebook sees invariant shadows: traces, norms, determinants, discriminants, coefficients, holonomies, periods, spectra, or measurement shares.

Thus:

public invariant shadows are what finite codebooks can read when hidden channels do not split.

This is another form of conserved relationality. The public world does not see the channels directly, but it sees invariant reports of their hidden structure.

A prime archetype, in this language, is an unsplit channel grammar whose invariant public shadows cannot be generated by lower viable grammars. That doctrine is developed elsewhere. Here it enters only as one descendant of stable finite reentry: if hidden structure cannot be directly quoted, it may still return through invariants.

8.5 Dark remainder

A finite public world also fails to surface all held support. Some support is not matter-capable in the public grammar. Some is matter-capable but unsurfaced. Some surfaces as public matter or messenger structure.

At the projector level, this appears as a staged chain

$$\Pi_b \le \Pi_{\mathrm{cap}} \le I$$

and an associated graded decomposition

$$I = (I - \Pi_{\mathrm{cap}}) + (\Pi_{\mathrm{cap}} - \Pi_b) + \Pi_b.$$

This is not needed for the main theorem, but it illustrates the general principle:

dark sectors are public-world remainders of finite staging.

They are not primitive substances of the deep. They are public renderings of what finite public form cannot surface in the same way as ordinary matter.

8.6 Semantic surprise and codebook extension

In an inward semantic atlas, a meaning is a gluable family of semantic contents or distinction projectors. If a context loop returns a distinction altered, the result is semantic surprise:

$$\mathfrak{S}_\gamma^{\mathrm{sem}}(\Pi) = \left\| T_\gamma \Pi T_\gamma^* - \Pi \right\|_{\mathrm{HS}}.$$

Persistent semantic nonclosure may force codebook extension. A concept that could previously be read only through shadows becomes understood when the mind-like custodian acquires the projectors needed to split it.

Thus:

understanding is codebook extension driven by persistent nonclosure.

Again, the paper does not need the full semantic mechanics. It records the common pattern:

$$\text{return} \longrightarrow \text{failure} \longrightarrow \text{correction} \longrightarrow \text{new public form.}$$

8.7 Summary table

Failure mode	Formal report	Public descendant
Carrier misalignment	$[W, P] \neq 0$	burden / mass-like cost
Carrier-basis coherence	off-diagonal $P_{\alpha\beta}$	dephasing / heat
Projector loop defect	$\mathfrak{S}_\gamma(\Pi) > 0$	holonomy / force
Unsplit channel system	no public splitting projectors	invariant shadows
Staged nonsurfacing	associated graded remainders	dark-sector rendering
Semantic loop defect	inward holonomy mismatch	surprise / reinterpretation
Persistent nonclosure	failed existing codebook	codebook extension

The common law is:

finite public form cannot violate conservation of relationality for free.

8.8 Summary

Failed return is not one thing. It becomes burden when participation is sharpened but carrier-misaligned. It becomes heat when carrier-incompatible coherence is dephased. It becomes holonomy when public projectors fail around loops. It becomes invariant shadow when hidden channels cannot split. It becomes dark remainder when held support cannot surface. It becomes semantic surprise when inward meanings fail to return. The single source is finite participation trying to quote exact relational conservation without perfect alignment.

9 Conservation of relationality

Exact carrierhood conserves relationality. Finite public worlds quote relationality. Nonclosure is the tax of the quote.

We can now state the central principle in its compact form.

Definition 9.1 (Relationality, working form)**.** In the analytic setting of this paper, *relationality* is the comparison structure encoded by the form

$$\mathcal{E}(f, g) = \langle Wf, Wg \rangle .$$

This definition is deliberately local and analytic. It does not claim that relationality is exhausted by one Hilbert-space comparison form. It states the form in which relationality becomes mathematically visible in a given comparison regime.

Theorem 9.2 (Conservation of relationality)**.** *Exact continuation by the first-order carrier preserves relationality:*

$$\mathcal{E}(\mathcal{F}_s f, \mathcal{F}_s g) = \mathcal{E}(f, g).$$

Proof. This is Theorem 4.1. □

The nontrivial part of the doctrine is not the unitary identity by itself. The nontrivial part is what happens when finite public form tries to quote the conserved relationality.

A finite cut holds a quote

$$\mathbb{O}[\chi],$$

which descends to finite participation

$$\mathbb{P}[\chi] = \mathrm{Norm}_{[0,1]}(\mathbb{O}[\chi]) .$$

Generic participation is written

$$0 \leq P \leq I.$$

Finite participation is freely returnable only when

$$[W, P] = 0.$$

Otherwise it pays:

$$\mathfrak{N}_W(P) = \frac{1}{2} \, \|[W, P]\|_{\mathrm{HS}}^2 .$$

Thus:

the conserved object is relationality; the tax is nonclosure of finite participation.

9.1 No free distinguishability

The conservation of relationality implies a principle of no free distinguishability.

A finite public world may:

hold, average, sharpen, dephase, split, glue, rebuild, extend its codebook.

But it does not create distinguishability from nothing. It reexpresses distinguishability under finite constraints.

We can state the principle informally as:

finite quotes expose, compress, or tax relationality; they do not create it ex nihilo.

This is not a claim about primitive substance. It is a structural claim about comparison. A public object is a sharpened quote of comparison structure. A public law is a stable invariant of such quotation. A public force is correction pressure from failed return. A public meaning is inward gluing of distinction.

9.2 Stable return as generatorhood

A generator is not simply what appears first in time. A generator is what remains non-erasable under lawful finite return.

Working definition:

generatorhood is stable return under admissible finite quotation.

More explicitly, a candidate generator is a distinction or channel whose finite quotes:

1. survive hold,
2. admit participation,
3. remain identifiable under continuation,
4. sharpen into public codewords or stable invariant shadows,
5. cannot be removed without changing the world that carries them.

This is the bridge from Gradiance to public law. A law is not primitive furniture. A law is stable public testimony about non-erased return.

law is compressed testimony of stable return.

9.3 Why exact return is not optional

A finite world does not have to orbit stably. It can fail. But if it fails too strongly, the consequences are direct.

If distinctions cannot return, there is no stable objecthood.

If participation cannot sharpen, there is no public codeword.

If projectors cannot glue, there is no public stage.

If carrier mismatch is too large, there is no low-burden matter or messenger regime.

If loop holonomy cannot be corrected, the public model fractures.

If semantic nonclosure cannot be metabolized, meanings fail to integrate.

Thus stable finite reentry is not an aesthetic preference. It is the condition of public form.

a regime may fail to return stably; but then it fails to become a public world.

Conservation of relationality is therefore not merely a law inside a world. It is a condition for a world to have stable public ledgers at all.

9.4 Matter as taxed relationality

Matter is often treated as primitive substance. In this framework, matter is a late regime.

A matter-like public object is finite participation that has sharpened enough to be object-like but still pays nonzero return tax:

$$\delta_{\mathrm{sh}}(P) \ll 1, \qquad \mathfrak{N}_W(P) > 0.$$

Thus:

matter is sharpened, glued, burdened relationality.

A messenger-like object is sharpened, glued, low-tax relationality:

$$\delta_{\mathrm{sh}}(P) \ll 1, \qquad \mathfrak{N}_W(P) \approx 0.$$

This does not by itself derive the full particle table. It gives the structural distinction beneath it:

matter carries taxed return; messengerhood carries low-tax return.

9.5 The conservation law in one diagram

The doctrine can be compressed as:

$$\begin{gathered}
\text{possible distinguishability} \\
\Downarrow \\
\text{comparison form } \mathcal{E} \\
\Downarrow \\
\text{carrier } W = L^{1/2} \\
\Downarrow \\
\text{exact return } \mathcal{F}_s = e^{-isW} \\
\Downarrow \\
\text{conserved relationality} \\
\Downarrow \\
\text{held quote } \mathbb{O}[\chi] \\
\Downarrow \\
\text{finite participation } \mathbb{P}[\chi] \\
\Downarrow \\
\text{sharpening and projector extraction} \\
\Downarrow \\
\text{return tax } \mathfrak{N}_W(\mathbb{P}[\chi]) \\
\Downarrow \\
\text{gluing and public worldhood.}
\end{gathered}$$

Everything downstream is a public fate of this sequence.

9.6 Summary

Exact continuation preserves the comparison form. Finite public worlds do not hold that conservation directly as objecthood. They hold quotes, derive participation, sharpen projectors, pay return tax, and glue across cuts. Conservation of relationality is therefore the exact law; public form is the finite, sharpened, and sometimes taxed report of that law.

10 Consequences and descendants

The conservation law is small. Its descendants are large. Once finite return is taxed, the major phenomena of public worlds become different reports of the same constraint.

This section records, without developing in full, the main descendant readings of conservation of relationality.

10.1 Gauge and transport

A transport mode is messenger-like when it preserves the relevant finite participation or public stage. Algebraically, for a generator X,

$$[X, P] = 0$$

means X preserves the finite participation state.

The stabilizer algebra is

$$\mathrm{Stab}(P) = \{X : [X, P] = 0\}.$$

Broken or burdened modes are those for which

$$[X, P] \neq 0.$$

Thus:

gauge symmetry is the transport grammar of preserved finite participation.

This is a direct descendant of conservation of relationality. A transport mode is free only when it preserves the relevant relation-quote.

At a cut χ, this is read with

$$P = \mathbb{P}[\chi].$$

10.2 Stage response

Let S be a public stage with projector Π_S. Its hidden response under continuation is

$$R_S = (I - \Pi_S) W \Pi_S.$$

The public burden operator is

$$\mathcal{B}_S = R_S^* R_S,$$

and the hidden response operator is

$$\Xi_S = R_S R_S^*.$$

The nonzero spectra of $\mathcal{B}_S$ and Ξ_S coincide. Public burden and hidden response are singular partners of the same stage-to-hidden map.

Thus:

public leakage and hidden response are the same nonclosure seen from opposite sides.

This is the stage-level form of the return tax.

10.3 Prime archetypes and public shadows

If a channel system cannot be split by the public codebook, its hidden channels may still return through invariant shadows. The public codebook may see traces, norms, coefficients, discriminants, holonomy traces, measurement shares, periods, spectra, or stable semantic consequences.

Thus:

public invariant shadows are finite-codebook reports of hidden conserved relationality.

A prime archetype is an unsplit channel grammar whose public invariant shadows cannot be generated by lower viable grammars. In this paper, that doctrine appears only as a descendant of no free distinguishability.

The shared pattern is:

$$\text{hidden channel} \longrightarrow \text{unsplit public codebook} \longrightarrow \text{invariant shadow} \longrightarrow \text{possible codebook extension.}$$

10.4 Dark remainders

Finite public staging does not surface all held support. The associated graded of nested public gates produces remainders. Such remainders are not failures of reality. They are public-world categories of nonsurfacing.

A typical two-gate staging is

$$\Pi_b \leq \Pi_{\text{cap}} \leq I,$$

with associated graded decomposition

$$I = (I - \Pi_{\text{cap}}) + (\Pi_{\text{cap}} - \Pi_b) + \Pi_b.$$

Thus:

dark sectors are finite-stage remainders of conserved relationality under public rendering.

The public world does not destroy the unsurfaced relational structure. It renders it as closure burden, hidden support, or nonsurfaced matter-capability.

10.5 Meaning

Inwardly, the same principle becomes Semantic Mechanics.

A meaning is a gluable family of semantic contents or distinction projectors. It is meaningful when it can return across internal contexts without dissolving. Semantic surprise is holonomic nonclosure. Understanding is codebook extension that reduces persistent semantic nonclosure.

Thus:

meaning is conserved relationality inside an inward atlas.

A mind-like system is a recursive custodian of inward finite reentry. It holds semantic quotes, maintains participation, sharpens meanings, pays nonclosure, and extends its codebook when return fails.

10.6 Matter, meaning, and law

Matter, meaning, and law are different public fates of stable finite reentry.

Matter is finite participation that sharpens and pays return tax.

Meaning is inward participation that glues across contexts and survives return.

Law is public compression of stable return.

Thus:

$$\begin{array}{ccl} \text{matter} & : & \text{burdened public return,} \\ \text{meaning} & : & \text{inward gluable return,} \\ \text{law} & : & \text{compressed testimony of return.} \end{array}$$

These are not the same object. But they share the same backbone:

$$\text{finite distinction} \longrightarrow \text{return} \longrightarrow \text{nonclosure or conservation} \longrightarrow \text{public role.}$$

10.7 The shared pattern

Across these descendants, the same pattern repeats:

$$\begin{array}{c} \text{exact carrier structure} \\ \Downarrow \\ \text{finite held quote} \\ \Downarrow \\ \text{finite participation} \\ \Downarrow \\ \text{return test} \\ \Downarrow \\ \text{nonclosure tax} \\ \Downarrow \\ \text{public descendant.} \end{array}$$

This is why one small conservation principle has broad explanatory reach.

10.8 Summary

Gauge symmetry, stage response, public shadows, dark remainders, matter-like burden, semantic surprise, and codebook extension are not unrelated ideas. They are descendants of one structure: exact carrierhood preserves relationality, while finite participation pays for misalignment.

11 Conclusion: no free return

A finite world is not made first of objects. It is made of distinguishability that can return. Exact carrierhood conserves the comparison structure. Public form is the sharpened and taxed part of that conservation.

This paper isolated a conservation principle beneath ordinary conserved quantities.

The first analytic object is a comparison form

$$\mathcal{E}.$$

It induces a nonnegative burden operator

$$L$$

and a first-order carrier

$$W = L^{1/2}.$$

Exact continuation

$$\mathcal{F}_s = e^{-isW}$$

preserves comparison:

$$\mathcal{E}(\mathcal{F}_s f, \mathcal{F}_s g) = \mathcal{E}(f, g).$$

This is conservation of relationality in analytic form.

Finite public worlds do not hold exact carrierhood directly. A cut first holds a quote

$$\mathbb{O}[\chi].$$

That quote descends to participation:

$$\mathbb{P}[\chi] = \mathrm{Norm}_{[0,1]}(\mathbb{O}[\chi]), \qquad 0 \leq \mathbb{P}[\chi] \leq I.$$

Finite participation becomes public only by sharpening toward projectorhood:

$$\Pi[\mathbb{P}[\chi]] = \mathbf{1}_{(1/2,1]}\big(\mathbb{P}[\chi]\big).$$

Its sharpness defect

$$\delta_{\mathrm{sh}}(\mathbb{P}[\chi]) = \mathrm{tr}\big(\mathbb{P}[\chi] - \mathbb{P}[\chi]^2\big)$$

controls distance to public projectorhood.

But public thinghood is not enough. Participation must also return under continuation. Its failure is the return tax:

$$\mathfrak{N}_W(\mathbb{P}[\chi]) = \frac{1}{2}\,\|[W, \mathbb{P}[\chi]]\|_{\mathrm{HS}}^2.$$

For a generic finite participation state

$$P = \sum_a \lambda_a \Pi_a,$$

the exact spectral identity

$$\mathfrak{N}_W(P) = \sum_{a<b} (\lambda_a - \lambda_b)^2\,\|\Pi_a W \Pi_b\|_{\mathrm{HS}}^2$$

shows that burden appears precisely where continuation crosses unequally held sectors.

A local projector is not yet a public world. Publicity requires gluing across cuts:

$$\Pi[\mathbb{P}[\chi']] \approx T_{\chi\to\chi'} \Pi[\mathbb{P}[\chi]] T^*_{\chi\to\chi'}.$$

Loop mismatch becomes holonomy. Holonomy induces correction pressure. Persistent nonclosure forces either dephasing, rebuild, codebook extension, or new public form.

The full doctrine is:

exact carrierhood conserves relationality;

finite public worlds quote relationality;

nonclosure is the tax of finite participation.

This is the sense in which there is no free return. A finite public world may expose, sharpen, average, hide, split, or reexpress relationality. It may not create distinguishability from nothing. Stable public form exists only where distinction can return recognizably or where the cost of failed return can be carried.

Matter, in this framework, is not primitive substance. It is sharpened, glued, burdened relationality. Messengerhood is sharpened, glued, low-burden relationality. Dark sectors are public remainders of finite staging. Gauge curvature is transport nonclosure. Meaning is inward public gluing. Understanding is codebook extension.

The final compression is:

$$\begin{gathered}\text{Gradiance}\\ \Downarrow\\ \text{comparison}\\ \Downarrow\\ \text{carrier}\\ \Downarrow\\ \text{conserved relationality}\\ \Downarrow\\ \text{held quote}\\ \Downarrow\\ \text{finite participation}\\ \Downarrow\\ \text{sharpening}\\ \Downarrow\\ \text{return tax}\\ \Downarrow\\ \text{public world.}\end{gathered}$$

The deep does not hand finite worlds objects. It gives them relationality. A world appears where that relationality can be held, admitted into participation, sharpened, carried, and returned. Conservation of relationality is the deep law. Public reality is its finite, taxed, gluable surface.

A Formula sheet and status ledger

This appendix collects the main formulas and records the status of the paper's claims.

A.1 Canonical ladder

The deep object is

$$\mathbb{O}.$$

A finite cut holds

$$\mathbb{O}[\chi].$$

Participation is

$$\mathbb{P}[\chi] = \mathrm{Norm}_{[0,1]}(\mathbb{O}[\chi])\,, \qquad 0 \le \mathbb{P}[\chi] \le I.$$

The public projector is

$$\Pi[\mathbb{P}[\chi]] = \mathbf{1}_{(1/2,1]}\big(\mathbb{P}[\chi]\big).$$

The canonical ladder is

$$\mathbb{O}[\chi] \longrightarrow \mathbb{P}[\chi] \longrightarrow \Pi[\mathbb{P}[\chi]].$$

A.2 Carrier formulas

Comparison form:

$$\mathcal{E}(f, g) = \langle Wf, Wg\rangle\,.$$

Burden and carrier:

$$W = L^{1/2}.$$

Holding:

$$\mathcal{P}_\sigma = e^{-\sigma W}.$$

Exact continuation:

$$\mathcal{F}_s = e^{-isW}.$$

Rebuild:

$$\mathcal{H}_r = e^{-rW^2}.$$

Conservation:

$$\mathcal{E}(\mathcal{F}_s f, \mathcal{F}_s g) = \mathcal{E}(f, g).$$

A.3 Participation diagnostics

For generic finite participation

$$0 \le P \le I,$$

total participation:

$$\mathrm{part}(P) = \mathrm{tr}\, P.$$

Sharpness defect:

$$\delta_{\mathrm{sh}}(P) = \mathrm{tr}(P - P^2).$$

Public projector:

$$\Pi[P] = \mathbf{1}_{(1/2,1]}(P).$$

Sharpness bound:

$$\|P - \Pi[P]\|_{\mathrm{HS}}^2 \le \delta_{\mathrm{sh}}(P).$$

Return tax:

$$\mathfrak{N}_W(P) = \frac{1}{2} \|[W, P]\|_{\mathrm{HS}}^2 .$$

Spectral tax identity:

$$P = \sum_a \lambda_a \Pi_a \quad \Longrightarrow \quad \mathfrak{N}_W(P) = \sum_{a<b} (\lambda_a - \lambda_b)^2 \|\Pi_a W \Pi_b\|_{\mathrm{HS}}^2 .$$

Carrier-basis identity:

$$\mathfrak{N}_W(P) = \frac{1}{2} \sum_{\alpha,\beta} (\omega_\alpha - \omega_\beta)^2 |P_{\alpha\beta}|^2.$$

A.4 Gluing formulas

Full public gluing:

$$\Pi[\mathbb{P}[\chi']] \approx T_{\chi\to\chi'} \Pi[\mathbb{P}[\chi]] T_{\chi\to\chi'}^*.$$

Shorthand:

$$\Pi_\chi := \Pi[\mathbb{P}[\chi]].$$

Compact gluing:

$$\Pi_{\chi'} \approx T_{\chi\to\chi'} \Pi_\chi T_{\chi\to\chi'}^*.$$

Loop defect:

$$\mathfrak{S}_\gamma(\Pi) = \left\| T_\gamma \Pi T_\gamma^* - \Pi \right\|_{\mathrm{HS}} .$$

Projected curvature:

$$F_S = \Pi_S (d\Pi_S \wedge d\Pi_S) \Pi_S.$$

A.5 Exact theorem-level statements

The paper uses or proves the following exact results:

1. closed nonnegative comparison forms induce nonnegative self-adjoint burden operators;
2. exact continuation by e^{-isW} preserves the comparison form;
3. commutant observables are conserved under exact continuation;
4. $\Pi[P] = \mathbf{1}_{(1/2,1]}(P)$ is a Hilbert–Schmidt nearest projector to P;
5. $\|P - \Pi[P]\|_{\mathrm{HS}}^2 \le \delta_{\mathrm{sh}}(P)$;
6. the return-tax identity
$$\mathfrak{N}_W(P) = \sum_{a<b} (\lambda_a - \lambda_b)^2 \|\Pi_a W \Pi_b\|_{\mathrm{HS}}^2$$
holds for finite participation;
7. in a carrier eigenbasis,
$$\mathfrak{N}_W(P) = \frac{1}{2} \sum_{\alpha,\beta} (\omega_\alpha - \omega_\beta)^2 |P_{\alpha\beta}|^2;$$

8. dephasing by $e^{-t\,\mathrm{ad}_W^2}$ monotonically decreases $\mathfrak{N}_W$;
9. local gluing defects bound path gluing defects;
10. smooth projector curvature satisfies

$$F_S = \Pi_S(d\Pi_S \wedge d\Pi_S)\Pi_S.$$

A.6 Interpretive principles

The following are interpretive principles built on the exact formulas:

1. before conserved quantities, there is conserved relationality;
2. exact carrierhood conserves relationality;
3. finite participation pays the return tax;
4. matter-like behavior is sharpened finite participation with nonzero return tax;
5. messenger-like behavior is sharpened finite participation with low return tax;
6. force is correction pressure from public-stage nonclosure;
7. hidden invariant shadows are finite-codebook reports of unsplit channel structure;
8. dark sectors are finite-stage remainders of public rendering;
9. meaning is inward public gluing;
10. understanding is codebook extension.

A.7 Guardrails

The paper does not claim:

1. that the deep is exhausted by one Hilbert-space comparison form;
2. that held quotes are automatically participation states;
3. that participation states are automatically projectors;
4. that one local projector is a public world;
5. that all physical mass is derived here;
6. that the Standard Model representation package is derived here;
7. that dark-sector rendering is completed here;
8. that Semantic Mechanics is completed here.

The claim is narrower and sharper:

exact carrierhood conserves relationality,

and

finite public participation pays for misalignment with that conservation.